

Optimal Asset Allocation Using Predicting Stock and Coin outputs in the Iranian Capital Market

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Abstract

One of the most important factors in deciding on investment is the amount of risk and output on capital assets. Choosing a set of optimal assets is often done by exchanging between risk and output, the higher the risk, so investors expect higher outputs. Portfolio optimization is about choosing the best combination of assets to maximize output on investment and minimize risk as much as possible. Therefore, one of the important steps in portfolio formation is to determine the optimal ratio or weight of assets to reduce the risk of investment portfolio. This important step is made by choosing the right strategy. The present study investigates the optimal allocation of assets (coins and stocks) using macroeconomic variables. The purpose of this study is to compare the performance of a predictability-based portfolio with a strategy-based portfolio ($\frac{1}{N}$). The data of this study were collected from internet databases stock exchange and central bank of Iran. The data are collected monthly from the beginning of March 2001 until the end of March 2017. In order to form predictability-based portfolios, first, out-of-sample stock and coin return forecasts were made by recursive and rolling regression models by five (24, 48, 60, 90, 120) month windows regression models. Then, by comparing the predictive power of the models within the sample, the optimal model is selected to predict the next periodic output and to predict the output on both stocks and gold. Then, using the predicted output on assets per month and in each window, two investment portfolios based on the variance mean strategy (investment strategy in tangent portfolio and variance-mean investment strategy with 3 and 5 aversion coefficient and another portfolio based on the minimum variance strategy have been performed and the performance of each of these portfolios with equal weight portfolios has been investigated through means comparison test, variance comparison test and Sharp Ratio. The results of the comparative test of variances and the Sharp ratio showed that the strategy of mean variance with a specified risk aversion coefficient (three and five) in all windows was able to defeat the strategy ($\frac{1}{N}$). The reason for the better performance of the mean variance strategy is that the underlying decision making is the predictability of asset outputs, and the weighting of each asset is based on the projected maximum output per month. The weighting of each asset per month is based on the maximum expected output.

Keywords: Sharp Ratio, Equal Weight Method, Performance Evaluating Criteria, Asset Allocation.

1. Introduction

Studies have shown that about 40% of the portfolio's return on equity relates to asset allocation (meaning what weight of the investment should be given to each of its core assets including equity, gold, currency, bank deposits, and so on) and about 60% of portfolio outputs are related to the choice of securities or appropriate investment options in each asset class for investment (Ibbotson & Kaplan, 2000).

Given that risk-free assets (bank deposits) have definite outputs, investors investing in this type of asset are certain of their output on investment for the next period, but for investing in assets such as coins and gold, real estate and corporate stocks is a risky decision-making due to the uncertainty of the outputs level of its property. In this case, investors choose a combination of risky and risk-free assets to act cautiously, in which case the issue of asset allocation is raised.

One of the best definitions put forward by experts about asset allocation is: Distribution of capital among different types of assets such as cash, bonds, stocks, commodities, etc., which are optimized for the risk-output balance based on the specific institutional / individual investor position and goals. Therefore, asset allocation is a key concept in investment management. In other words, the process of dividing capital according to the investor's personal goals, risk taking and investment horizon, by balancing risk and output in different asset groups, is called asset allocation (Raei & Amir Hashemi, 2016). The most well-known asset allocation model is the mean-variance strategy (Modern Investment Portfolio Theory) first developed by Markowitz (1952) to optimize the process of capital allocation by assuming a fixed, investment opportunity set between different group of assets over a period of time. In fact, capital allocation is done among a number of existing assets in order to maximize the outputs on investment (Raei & Amir Hashemi, 1396).

Given that the outputs on assets such as stocks, coins, gold and other assets vary over time, the performance of markets and financial assets will be affected by the economical stagflation and activity, and consequently, the rate of output on assets changes or investor goals may change over the time which leads to portfolio changes. Under these conditions, proper portfolio management requires that the composition of the portfolio assets change according to the forecast, given the concept of predictability in the financial field. Therefore, if the investor can predict the outputs of different assets using the predictor variables, he can optimally allocate his property to these assets over time which results in synchronization of portfolios with business cycles in different sectors or industries.

To predict the outputs on assets, we must examine this capability over the time with respect to the specific predictor variables of each asset. That is, to use predictive variables that can update the portfolio as the economy evolves so that the selection of portfolio assets at the beginning of the investment period depends on the performance of the predictor variables, which is determine the weight of each asset for the specified investment period according to the defined model (in the method section). According to research results that have shown that macroeconomic variables affect the return on assets, therefore, their impact investments on assets enable the prediction of outputs on investment assets.

Also one of the issues that are studied in finance knowledge is whether the use of predictability leads to better performance of investment strategies or not. In the literature of modern portfolio theory, various variables are used to implement strategies. Given that in the stock market, corporate performance and asset market and in the financial asset, performance is affected by macroeconomic variables, therefore, one of the predictive options is to use macroeconomic variables to predict the outputs on the cash price index of stocks and assets. Tangjitprom (2012) has shown that macroeconomic variables used in empirical research can be divided into four groups: Variables related to general economic conditions, variables that include interest rates and monetary policy, variables reflecting the level of prices and variables related to international activities. There are many variables that can fall into this classification. The relationship between stock prices and macroeconomic variables has been well illustrated by stock valuation models. According to these models, the current stock price is approximately equal to the present value of future cash flows, hence any macroeconomic variables affect the cash flow and the expected rate of output in turn affects the amount of the share. It is believed that stock values are determined on the basis of some basic macroeconomic variables (Kirui, Wawire & Onono, 2014).

The macroeconomic variables selected in this study are: GDP, Inflation rate, Exchange Rate and petroleum which are used to predict stock and coin returns in the Iranian capital market. The purpose of this study is to predict the price index of cash outputs as a representative of the return on investment in corporate stocks and gold coin output based on the predictive variables of each of these two assets. Then the investor performance to determine the weight of the assets (ratio of risky and risk-free assets) comparing with the performance of investor that equally holds all three types of assets (risky and risk-free) measured using portfolio performance evaluation criteria.

Research hypotheses

1. Stock outputs on Tehran Stock Exchange are predictable.
2. Coin outputs are predictable in the Iranian capital market.
3. Determining the optimal weight of the mean-variance strategy by using the predictability of output on assets leads to better performance than the equal weighting strategy.

Economic variables used in the study

1. (Price Coin) (r_1)

In this study, the rate of the coin output is considered as the dependent variable. The rate of output on a coin is: The average of the monthly change percentage about selling price of all coins bahar azadi in Tehran's free market.

2. (Tedpix) (r_2)

The Price and Cash output Index or total income index (Tedpix), has been calculated and published since March 1998 in Tehran Stock Exchange. Changes in this index reflect the total output on the stock exchange and are affected by price changes and cash outputs. This index covers all companies listed on the exchange and its weighting and calculation method is similar to the Tedpix and the only difference between them is in their adjustment method.

Price index and cash output of Tehran Stock Exchange are calculated by formula (1-3).

$$\text{Tedpix}_t = \frac{\sum_{i=1}^n P_{it} Q_{it}}{RD_t} \times 100 \quad (1)$$

P_{it} : i company price at t time

Q_{it} : Number of shared assets by i company at t time

RD_t : The basis of the price index and the cash output at t time which was originally equal to $\sum p_{i0} q_{i0}$

The rate of output on the stock price index and the cash output on stocks are: Percentage of monthly change in the price index and the stock cash output.

3. Output rate of unit load Oil Price, APEC Oil Basket (oil)

The rate of output on unit load oil prices is the percentage of monthly average about prices of unit loads of crude oil.

4. Inflation rate (INF)

That is the continuous and pervasive rise in the general level of prices (Shakeri, 2008).

5. Growth Rate of (GDP)

The monetary value of all final goods and services produced within a country's borders over a given period of time (Shakeri, 2008). Since all variables are considered monthly in this study, the seasonal rate of GDP growth is converted to monthly rates through Eviews software.

6. Exchange Rate of Return (EX)

The exchange rate is the criterion for the equality of the money of a country against the money of other countries and it reflects the economic situation of that country compared to the economic conditions of other countries. In an open economy, the exchange rate is a key variable because of its interconnection with other domestic and foreign variables, both of which influence domestic and foreign policies and economic developments. Conversely, the exchange rate is a variable that can affect the performance of the economy and variables (Hallafi et al., 2004).

Exchange output rate is: Average percentage of monthly change about selling price of one US dollar in the Tehran Free Market.

In this study, the rate of output of the introduced variables is used. The rate of output is the percentage of change in the value of an asset, or the variable over a period (Suranovic, 2012).

$$(2) \text{ Rate of return} = \frac{R_t - R_{t-1}}{R_{t-1}} * 100$$

Research Methodology

The first step is how to predict expected returns on stocks and coins

This study examines the investment behavior that intends to select its investment weights from the three stocks, coins and risk-free assets (bank deposit rate of return). To this end, it tries to predict the next period for both coin and stock assets by using the predictability of the next period output and based on that predictability using its optimal investment weight using the Markowitz model and finally compare the strategy performance of this investment with the equity strategy that assigns each equity asset to any other asset without any optimization.

Similarly, in order to examine the extent to which predictability is effective in better performance of the investment strategy, the performance of the investor strategy is also performed using the minimum variance strategy. In order to predict the expected return on the two stocks and coins in the month following the macroeconomic variables including exchange rate, inflation rate, GDP growth rate, oil output rate and one-month output on coins and stocks as a precursor variable. The reason for selecting these variables is explained in Chapter 2, Sections 2-6. Naturally, due to the multitude of models that use expected outputs, it is possible to select 2^k models per month. In order to select the optimal model from the models implemented above, two criteria are based;

1. A model is chosen in which the coefficients of the predictor variables become significant.
2. From the above models, a model with the highest criterion \bar{R}^2 is selected.

Forecasts of expected outputs on stocks and coins are calculated through regression models in the form of exemplary forecasts. In order to perform extrapolated predictions the length of the period under study is divided into two parts: $T = n + q$ That is, n initial data is used to estimate the model and then predict q later data (Pesaran & Timmermann, 1996).

One of the important choices in extrapolation prediction is related to the period in which the regressions are estimated, so we use two methods of extrapolation prediction called recursive and rolling method.

The length of the period which is studied, evaluated both in the form of short meters long window estimation windows and in the form of long meters long window estimation windows.

- 1) 24 month window
- 2) 48 month window
- 3) 60 month window
- 4) 90 month window
- 5) 120 month window

Recursive Method

This method uses more data because the length of the period is increasing. To implement this method, an S meters long-window is first determined for the regression estimation, then all the estimation window data is used to predict the expected output for the next period. The process of predicting outputs is in the form of a relationship (3-3) (Zivot, 2011).

$$[1, \dots, S, \dots, S + h] \tag{3}$$

$$[1, \dots, S + 1, \dots, S + h + 1]$$

⋮

$$[1, \dots, S + N, \dots, S + h + N]$$

In this study, the recursive method is implemented in the introduced windows. For example, in a 24-month window, 24-month data is used to predict the expected output for the 25th month and to predict the expected output for the

26th month and data from 25 months ago are used and this process continues to predict expected returns each month until the end of the sample.

Rolling Method

The rolling method uses less data because the estimation window length does not increase and is constant, but at each stage of the forecast window the estimation period moves forward. A prominent feature of this method is that later data are used to predict expected outputs. To implement this method, the length of the estimation window is first determined. Then all the estimated window data is used to predict the expected output for the next period. The process of rolling prediction is in the form of a relationship (3-4) (Zivot, 2011).

$$[1, \dots, S, \dots, S + h] \quad (4)$$

$$[2, \dots, S + 1, \dots, S + h + 1]$$

$$[N, \dots, S + N, \dots, S + h + N]$$

In this study, the rolling method is also implemented in all the introduced windows. For example, the 24-month forecast window uses 24-month data to predict the expected 25th month output, and then uses the past 24-month data only to predict the 26th month output. This is because the length of the estimation window is constant but the estimation window is rolling and moving forward at any time. This process continues until the end of the sample to predict the expected output each month. In this study, at each stage of the forecast window, the estimation window moves forward one month.

The research model in the first step

After selecting the forecasting methods and estimating the estimation windows, the expected output on stocks and coins per month is estimated by linear regression models through Equation (3-5). Since six independent variables are used in this study to estimate expected outputs on stocks and coins, the regression model is estimated at 2^6 per month. That is, 64 models are estimated per month and 12288 models are estimated for the whole sample.

$$\tau = 1, 2, \dots, t - 1 M_i: \rho_{\tau+1} = \beta_i X_{\tau,i} + \varepsilon_{\tau+1,i} \quad (5)$$

In equation (3-5), $i : M_i$ is model and $X_{\tau,i} : (k_i + 1) \times 1$ is the vector of the regressors entered into the model. The parameters of the M_i model are estimated by the ordinary least squares estimation (ols) method in Equation (6):

$$\hat{\beta}_{t,i} = \left(\sum_{\tau=0}^{t-1} X_{\tau,i} X'_{\tau,i} \right)^{-1} \sum_{\tau=0}^{t-1} X_{\tau,i} \rho_{\tau+1} \quad t = k + 2, k + 3, \dots, T \quad i = 1, \dots, 2^k \quad (6)$$

Then, the AIC, SC, R² criteria are used to select the best regression model from the estimated models each month. These criteria are obtained from (9), (10) and (13) relationships, respectively. To calculate the Akaike criterion, the maximum likelihood function is first calculated by Equation (3-7).

$$\widehat{LL}_{t,i} = \frac{-t}{2} \{1 + \log(2\pi \hat{\sigma}_{t,i}^2)\} \quad (7)$$

$$\hat{\sigma}_{t,i}^2 = \sum_{\tau=0}^{t-1} (\rho_{\tau+1} - X'_{\tau,i} \hat{\beta}_{t,i})^2 / t \quad (8)$$

$$AIC_{t,i} = \widehat{LL}_{t,i} - (K_i + 1) \quad \text{Akaike criterion} \quad (9)$$

$$\bar{R}^2_{t,i} = 1 - \frac{\hat{\sigma}_{t,i}^2}{S_{\rho,t}^2} \quad (\text{mediate}) \quad R^2 \quad (10)$$

$\tilde{\sigma}_{t,i}^2$ is the unbiased estimator from $\sigma_{t,i}^2$, which is calculated by Equation (3-11)

$$\tilde{\sigma}_{t,i}^2 = \sum_{\tau=0}^{t-1} (\rho_{\tau+1} - X'_{\tau,i} \hat{\beta}_{t,i})^2 / (t - k_i - 1) \quad (11)$$

$S_{\rho,t}^2 = \sum_{\tau=1}^t \frac{(\rho_{\tau} - \bar{\rho}_t)^2}{(t-1)}$ The sample variance for t observation is ρ and $\bar{\rho}_t$ is calculated according to the equation (3-12):

$$\bar{\rho}_t = t^{-1} \sum_{\tau=1}^t \rho_{\tau} \quad (12)$$

$$SC_{t,i} = \frac{1}{t} \sum_{\tau=1}^t \left\{ I(\rho_{\tau}) I(\hat{\rho}_{\tau,i}) + (1 - I(\rho_{\tau})) (1 - I(\hat{\rho}_{\tau,i})) \right\} \quad \text{Schwarz criterion} \quad (13)$$

Then, by selecting the regression models that had the highest \bar{R}^2 and at the same time the lowest AIC and SC, a time series of expected yields of the models with the maximum \bar{R}^2 was formed. It is then calculated to determine the severity of the relationship and the type of relationship (direct or inverse) between the expected output and the realized output each month through the relationship (14).

$$\text{corr}(X, Y) = \frac{(X, Y)}{\sigma_x \sigma_y} = \frac{E[(X - \mu_x)(Y - \mu_y)]}{\sigma_x \sigma_y} \quad (14)$$

Second step, describe the optimal asset allocation strategies

At this stage, we predict the weight of assets (stocks and coins) for the next month. First, using the expected returns per window in the previous step, the excess expected return in each window are calculated. Incremental output means the difference between realized or expected output on risk-free output. In the following, the average vector of the incremental outputs which is realized calculates. Through this vector and the realized for two assets of stocks and coins, the covariance variance matrix is formed and multiply by the expected incremental outputs and the weight of the two risk assets of the stock and the coin is determined each month.

For example, in the 24-month window, to predict the weight of two stocks and coins in the 25th month, first the expected incremental output is calculated in the 25th month, then the average of the incremental outputs calculated and the incremental output vector is multiplied by the preceding months and the risk weighted assets are weighted each month. Then investment portfolios based on these weights are formed each month and their performance is compared with the assets portfolios. Each month, three portfolios are formed based on the mean variance strategy with determined and tangent aversion coefficients. In addition, portfolios based on the strategy of minimum variance are formed each month.

Mean variance strategy

To execute the strategy of mean variance with a specific risk aversion coefficient ($\gamma = 3, 5$), an investment portfolio consists of two risk assets (stocks and coins) forms. The utility function of the investment is maximized by the relation (15):

$$\max_{X_t} X_t^S \mu_t - \frac{\gamma}{2} X_t^S \sum_t^T X_t \quad (15)$$

S represents the length of the window under consideration. T represents the total length of the time series. X_t Represents the portfolio weighted vector invested in the risk asset. The present study invests two stock risk and coin risk assets and X_t represents the c weighted vector.

Then, using the expected outputs of the two stocks and the coin vector, the incremental returns are formed. In this study, the risk-free rate of output is calculated 1.5. μ is the incremental return vector of the two stocks and coins according to the relation (16).

$$\mu = \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \quad (16)$$

Then, by the relation (17) the weight of assets (coins, stocks) is calculated at the end of each month.

$$W_t = \left(\frac{1}{\gamma} \right) \sum_t^{-1} \mu \quad (17)$$

In relation (17), sigma is equal to the covariance variance matrix of expected returns on stocks and coins and is calculated according to (18) relationship.

$$\sum = \begin{bmatrix} var_{r_1} & cov_{r_1 r_2} \\ cov_{r_2 r_1} & var_{r_2} \end{bmatrix} \quad (18)$$

In relation (17) γ is the risk aversion factor. In this study, risk aversion coefficient is considered as 3 and 5. The risk aversion coefficient depends on the nature of the risk aversion, the higher the risk aversion, the more risk averse individuals are and the more willing to hold risk-free assets, and the lower the risk aversion coefficient, there are more risky individuals. The weighted vector of assets (stocks and stocks) can also be represented by the relation (19).

$$\begin{bmatrix} W_1 \\ W_2 \end{bmatrix} = \frac{1}{\gamma} \times \begin{bmatrix} var_{r_1} & cov_{r_1 r_2} \\ cov_{r_2 r_1} & var_{r_2} \end{bmatrix}^{-1} \times \begin{bmatrix} \mu_1 \\ \mu_2 \end{bmatrix} \quad \text{بردار وزنی دارایی ها} \quad (19)$$

Using the asset return vector in the first month of each window, we obtain the relationship (20-20).

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,s} & r_{1,s+1} & r_{1,s+2} & \dots & r_{1,2} \\ r_{2,1} & r_{2,2} & \dots & r_{1,s} & r_{2,s+1} & r_{2,s+2} & \dots & r_{2,T} \end{bmatrix} \quad \begin{array}{l} \text{Stock and Coin Return Vector (20)} \\ \text{in the first month} \end{array}$$

Also, the stock and coin return vector in the recursive and rolling windows is obtained according to the relations (21) and (22).

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,s} & r_{1,s+1} & r_{1,s+2} & \dots & r_{1,2} \\ r_{2,1} & r_{2,2} & \dots & r_{1,s} & r_{2,s+1} & r_{2,s+2} & \dots & r_{2,T} \end{bmatrix} \quad \begin{array}{l} \text{Stock and coin return vector (21)} \\ \text{for the second month in the} \\ \text{recursive window} \end{array}$$

$$R = \begin{bmatrix} r_{1,1} & r_{1,2} & \dots & r_{1,s} & r_{1,s+1} & r_{1,s+2} & \dots & r_{1,2} \\ r_{2,1} & r_{2,2} & \dots & r_{1,s} & r_{2,s+1} & r_{2,s+2} & \dots & r_{2,T} \end{bmatrix} \quad \begin{array}{l} \text{Stock and coin return vectors for the (22)} \\ \text{second month in a rolling window} \end{array}$$

The mean variance strategy is also implemented in the form of tangent portfolios. Tangent portfolio is the portfolio that tangents to the performance boundary. In this type of portfolio only risk assets are held. In the tangible portfolio, the total risk weight of the asset is equal to one, and to calculate the weight of an asset, its weight is divided by the total weight of the asset. In fact, after calculating weights, we normalize them. The weight of risky assets in the tangent portfolio is obtained through the relationship (23).

$$W_{\text{tan},t} = \frac{\frac{1}{\gamma} \sum_t^{-1} \mu_t}{\left| \frac{1}{\gamma} 1'_N \sum_t^{-1} \mu_t \right|} = \frac{\sum_t^{-1} \mu_t}{\left| 1'_N \sum_t^{-1} \mu_t \right|} \quad (23)$$

Minimum variance strategy

In this strategy, the optimal portfolio weights are more sensitive to the estimation of the mean errors of the predicted returns than the estimates of the errors in the covariance variance matrix. Therefore, this strategy completely ignores the expected returns and only the covariance variance matrix is used to form the optimal weights of the assets. (it means the basis of decision is only the covariance variance matrix) Minimum variance portfolio weights are obtained by solving the following equation (Demiguel, Garlappi & Uppal, 2007).

$$\min_{W_t} W_t^T \sum_t W_t \quad \text{S. t.} \quad 1_N^T W_t = 1. \quad (24)$$

$$W_{\text{minv},t} = \frac{\sum_t^{-1} 1_N}{1_N^T \sum_t^{-1} 1_N} \quad (25)$$

In this study, for the implementation of the minimum variance strategy in each window through the realized incremental returns in the past months, the covariance variance matrix of the returns on assets is computed each month and multiplied by the vector $1'_N$ and the weight of the assets. Identify the risks. Then, to calculate the minimum variance portfolio return on each month, the assigned weight of each asset is multiplied by its realized return.

Equal weighting strategy

An equal weighting strategy ($\frac{1}{N}$) involves holding assets in a weighting manner $W_t^{\text{ew}} = \left(\frac{1}{N}\right)$. This strategy is used for two reasons: First, the rule ($\frac{1}{N}$) is an easy measure to measure and implement because it ignores any optimization or estimation of the return on assets. Second, despite the theoretical models developed over the last 50 years and the advances in parameter estimation techniques, investors continue to use simple allocation rules for their wealth allocation (Demiguel, Garlappi & Uppal, 2007). In this study, we assign equal weight (0.5) to both stock and assets to form simple portfolio.

Methods for evaluating and comparing exemplary portfolio performance for implemented strategies

To compare the performance of each of the strategies implemented with the $(\frac{1}{N})$ strategy, the comparison of means, variances, and Sharp ratios is based on decision making. The null hypothesis in all three tests is that there is no significant difference with strategy $(\frac{1}{N})$.

Mean comparison test

To perform the test of comparing the mean strategies of the mean variance and the minimum variance, at first the mean of subtraction of for exemplary Strategy return from strategy return $(\frac{1}{N})$ is calculated and then tested whether the new series mean has significant difference with zero or not. For this purpose, the t-test is used according to the relationship (27). The hypotheses tested in this test are:

$$\begin{aligned} H_0 &= \mu_1^2 = \mu_2^2 \\ H_1 &= \mu_1^2 \neq \mu_2^2 \end{aligned} \tag{26}$$

$$t = \frac{\text{new series mean}}{\text{New Series Deviation Criterion}} \tag{27}$$

It is then examined by comparing the computational t and the value of t to reject or accept the null hypothesis.

Analysis of variance

In order to run the variance comparison test, the two-way ANOVA is used; the result is always a positive number because the variance is always positive. The following steps are performed for variance comparison test:

1. Hypotheses expression: This test examines the equality of variance between two societies.

$$\begin{aligned} H_0 &= \sigma_1^2 = \sigma_2^2 & \frac{\sigma_1^2}{\sigma_2^2} &= 1 \\ H_1 &= \sigma_1^2 \neq \sigma_2^2 & \frac{\sigma_1^2}{\sigma_2^2} &\neq 1 \end{aligned} \tag{28}$$

2. Calculate the critical value F: The variance of communities is divided; the larger variance is divided into smaller variance.

$$F = \frac{S_1^2/\sigma_1^2}{S_2^2/\sigma_2^2} \xrightarrow{\sigma_1^2 = \sigma_2^2} F = \frac{S_1^2}{S_2^2} \tag{29}$$

3. Calculation of the degree of freedom: equal to the sample size minus one

4. Considering the alpha level, in this study the alpha level is 0.05.

5. Compare the critical F with the F table

6. Reject or Accept the Hypothesis: If the calculated F value is greater than the F table, the null hypothesis can be rejected and vice versa.

To perform the test of variance comparison of mean variance strategies and minimum variance, first the variance of the considered strategy return and the variance of the strategy return are calculated $(\frac{1}{N})$. Then, using the F test, having or not significance of the null hypothesis based on not having a significant difference between the considering strategy variance and the strategy is computed $(\frac{1}{N})$. The F-test is calculated from Relation (29-23) (Azar, Momeni, p. 109).

Sharp Ratio Test

The Sharp Ratio Comparison Test is performed by computing the ZJK statistic presented by Jubson and Korky (1981). This statistic follows the standard normal distribution.

$$\hat{Z}_{JK} = \frac{\hat{\sigma}_N \hat{\mu}_i - \hat{\sigma}_i \hat{\mu}_N}{\sqrt{\hat{\theta}}} \tag{30}$$

$\hat{\vartheta}$ is obtained from relation (30). In this respect T is the sample size, M: the length of the investigated window. In this respect, $\hat{\sigma}_i \hat{\sigma}_n$ is the correlation of the strategy ($\frac{1}{N}$) and the calculated strategy (mean variance strategy with specific and tangent risk aversion coefficients, the minimum variance strategy) (Demiguel, Garlappi & Uppal, 2007).

$$\hat{\vartheta} = (2\hat{\sigma}_i^2 \hat{\sigma}_n^2 - 2\hat{\sigma}_i \hat{\sigma}_n \hat{\sigma}_{i,n} + \frac{1}{2} \hat{\mu}_i^2 \hat{\sigma}_n^2 + \frac{1}{2} \hat{\mu}_n^2 \hat{\sigma}_i^2 - \frac{\hat{\mu}_i \hat{\mu}_n}{\hat{\sigma}_i \hat{\sigma}_n} \hat{\sigma}_{i,n}^2) \frac{1}{T - M} \quad (31)$$

In this study, the significance of Sharp Ratio of Portfolios (based on Mean Variance Strategy and Minimum Variance Portfolios) was compared with Sharp Ratio of Equal Weight Strategy ($\frac{1}{N}$), and their significance was assessed through P-Value. Be. P-Value is the smallest amount of alpha level, which, according to the results of the sample, leads to the rejection of H_0 . Under these conditions, two different states arise As shown in relation (32).

$$\begin{cases} P - \text{Value} < \alpha & \text{فرض } H_0 \text{ رد می‌شود} \\ P - \text{Value} \geq \alpha & \text{فرض } H_0 \text{ رد نمی‌شود} \end{cases} \quad (32)$$

Research findings

Predictability of Stock and Coin Returns in Iran

Given the multiplicity of regressions (from univariate to six variables) and windows (5 recurse and 5 roll), the results are limited to univariate and two-variable rolling stock regressions in the 60-month window and regression. Univariate and Bivariate Coin Returns to the 48-month window.

Predictability of Stock Returns Predictability

In this section, the results of stock return predictability using six independent variables in the form of univariate roller regressions are presented.

Univariate rolling stock regressions

inflation	GDP	oil	Exchange rate	Gold return of last month	Property return of last mont	Roller Univariate regressions - 60 month window
144	144	144	144	144	144	Total number of predictions
4	41	73	0	24	108	Total number of significant regression
0/01	0/03	0/06	-0/01	0/01	0/11	Average of mediated \bar{R}^2

Table 1. Univariate rolling stock regressions
Source: Research findings

Table (1) shows the results of the rolling constant univariate regressions in the 60-month window. In the 60-month window, we predict 144-month returns. According to the mediated \bar{R}^2 average, which indicates that on average a few percent of the stock returns are related to the independent variables, in the univariate rolling regressions, the best variable predicting the stock returns is the one return is last month stock return. Also, according to the coefficient of significance test (t-test) out of 144 univariate regression coefficients of stock returns on return stock of last month, 108 items of regressions are significant. The second variable that is effective in predicting stock returns is oil return rates, and 73 of the univariate regression coefficients are significant. The third variable that is effective in univariate regressions in predicting stock returns is GDP, and 41 of the univariate rolling stock regressions are significant on GDP. Given the same average value of mediated \bar{R}^2 for the two variables inflation and gold return of last month, It can be said that there are effective in predicting stock returns equally, But given that there are more number of significant regressions on the gold return of last month, it is a priority over inflation to predict stock returns. Also the result from average of mediated \bar{R}^2 about variable of exchange rate shows that this variable does not have the power to predict stock returns.

Bivariate regression of stock returns

In this section, the results of predictability of stock returns are presented using six independent variables in the form of bivariate roller regressions.

stock and inflation returns of last month	stock and GDP returns of last month	stock and oil returns of last month	stock and exchange rate returns of last month	stock and gold returns of last month	Roller Bivariate regression - 60 month window
144	144	144	144	144	Total number of prediction
0	3	53	1	25	Total number of significant regression
0/11	0/12	0/14	0/1	0/13	Average of mediated \bar{R}^2
Oil and exchange rate	Gold and inflation of last month	Gold and GDP of last month	Gold and oil of last month	Gold and exchange rate of last month	Roller Bivariate regression - 60 month window
144	144	144	144	144	Total number of prediction
0	0	3	0	0	Total number of significant regression
0/05	0/02	0/04	0/08	0	Average of mediated \bar{R}^2
Inflation and GDP	Inflation and oil	GDP and oil	Inflation and exchange rate	GDP and exchange rate	Roller Bivariate regression - 60 month window
144	144	144	144	144	Total number of prediction
0	0	3	0	0	Total number of significant regression
0/02	0/07	0/09	-0/01	0/02	Average of mediated \bar{R}^2

Table 2. Bivariate regression of stock returns
Source: findings of the research

Table (2) shows the results of the bivariate regression coefficients in the 60-month window. The results show that in the bivariate regressions that include stock returns of last month and other variables (gold, oil, GDP, exchange and inflation of last month), have the higher average of \bar{R}^2 . Also each of these regressions has more number of significant regressions compared to bivariate regressions that included gold returns of last month and other variables or bivariate regressions that include only macroeconomic variables. Bivariate regressions, which are a combination of last month stock returns and other variables, are explained in greater detail due to the presence of the stock variable of last month in the stock returns regression. So they have higher average amount of \bar{R}^2 .

The results show that the rolling bivariate regression includes the stock and oil return of last month. The highest mean value of \bar{R}^2 and the highest number of significant regressions. Also, exchange rate regressions have no role in predicting stock returns.

Six-variable rolling stock regressions

last month stock return, last month gold return, exchange rate, oil, GDP, inflation	Six-variable rolling stock regressions- 60 month window
144	Total number of prediction
0	Total number of significant regression
0/15	Average of mediated \bar{R}^2

Table 3. Six-Roll Stock Returns Regressions

source: research findings

Table (3) shows that, on average, 0.15% of stock returns per month are explained by the above model (one month stock return, one month gold return, exchange rate, oil, GDP, inflation).

Predictability of Coin Returns investigation

Univariate Coin Returns Recursive Regression

Inflation	GDP	oil	Exchange rate	Gold returns of last month	Stock returns of last month	Univariate Coin Returns Recursive Regression- 48 month window
156	156	156	156	156	156	Total number of prediction
38	6	7	59	122	0	Total number of significant regression
0	0	0	0	0/13	-0/01	Average of mediated \bar{R}^2

Table 4- Univariate Coin Recursive Regression

Source: research findings

Table (4) shows the results of the univariate regression regressions in the 48-month window. This window is predicted to be 156 months. Given the amount of the average of mediated \bar{R}^2 , gold return of last month is the best variable to predict coin return. Amount of the average of mediated \bar{R}^2 , indicates that gold return of last month shows 0.13% of the coin return change on average. Also, 122 cases of univariate recursive regressions of gold return of last month have significant effect on coin return. the average of mediated \bar{R}^2 for univariate return regressions of gold a month ago in univariate recursive regressions of gold return of last month on exchange, oil, GDP and inflation indicates that none of these variables have the power to explain coin return changes. Therefore, it can be said that among the variables introduced, in the univariate recursive regression, only gold returns of last month can predict the coin return.

Bivariate regression coefficients of coin return

stock and inflation returns of last month	Stock and GDP returns of last mons	Stock and oil returns of last month	Stock and exchange returns of last month	Stock and gold returns of last month	Bivariate Recursive regressions - 48 month window
156	156	156	156	156	Total number of predictions
0	0	0	0	0	Total number of significant regression
-0/01	-0/01	-0/01	0	0/12	the average of mediated \bar{R}^2

Oil and exchange rate	Gold and inflation returns of last month	Gold and GDP returns of last month	Gold and oil return of last month	Gold and exchange rate returns of last month	Bivariate Rolling regressions - 60 month window
156	156	156	156	156	Total number of predictions
0	0	1	0	71	Total number of significant regression
0	0/12	0/12	0/12	0/13	the average of mediated \bar{R}^2
Inflation and GDP	Inflation and oil	GDP and oil	Inflation and exchange rate	GDP and exchange rate	Bivariate rolling regressions - 60 month window
156	156	156	156	156	Total number of predictions
32	0	0	43	0	Total number of significant regression
0/01	0	0	0/01	0	the average of mediated \bar{R}^2

Table 5. Bivariate Coin Recursive Regression

Source: research finding

Table (5) shows the results of the recursive bivariate regressions in the 48-month window. In bivariate regression including stock and gold returns of last month, average amount of \bar{R}^2 is reported to be 0.12% which means that coin return changes in every month shows by stock returns of last month and gold return of last month. In other bivariate regressions that include stock returns of last month and other variables (exchange, oil, GDP and inflation), the mean value of \bar{R}^2 is zero and negative indicating this type of regression are not capable of predicting coin returns. Given the mean value of \bar{R}^2 in bivariate regressions that included gold returns of last month and variables (exchange, oil, GDP and inflation) these regressions can predict the coin return, but in this window, none of these regressions coefficients was significant. Also, bivariate regressions including exchange rate and inflation, bivariate regressions of inflation and GDP shows 0.01% of the coin returns changes in the pyramid. In these two regressions, since the number of significant regressions is higher in inflation and exchange rate regressions. In relation to the bivariate regressions, inflation and GDP are prioritized.

Six-variable Coin Returns Regression

stock return of last month, gold return of last month, exchange rate, oil, GDP, inflation	Six-variable rolling regression of coin returns - 48 month window
156	Total number of predictions
0	Total number of significant regressions
0/12	the average of mediated \bar{R}^2

Table 6. Six-variable Coin Recursive Regression

Source: research findings

Table (6) shows, on average, 0.12% of the coin returns change per month as explained by the above model (stock return of last month, gold return of last, exchange rate, oil, GDP, inflation). Can be optimized portfolio results

Optimized portfolio results

Optimized portfolios with an incremental recursive estimation period

120 th window	90 th window	60 th window	48 th window	24 th window		Strategies
1/76	1/99	1/89	1/81	2	Total mean	Equal weight
4/74	4/52	4/26	4/22	4/27	Standard deviation	
0/37	0/44	0/44	0/43	0/47	Sharp ratio	
1/64	1/65	1/62	1/62	1/56	Total mean	Variance mean $\gamma = 3$
0/6	0/74	0/73	0/7	0/43	Standard deviation	
2/73	2/23	2/21	2/30	3/66	Sharp ratio	
1/59	1/6	1/58	1/57	1/54	Total mean	Variance mean $\gamma = 5$
0/40	0/49	0/49	0/43	0/26	Standard deviation	
3/97	3/24	3/24	3/68	5/94	Sharp ratio	
1/94	2/08	1/98	1/93	2/07	Total mean	Minimum variance
5/21	4/75	4/55	4/5	4/37	Standard deviation	
0/37	0/43	0/43	0/42	0/47	Sharp ratio	
6/52	2/98	-4/76	-2/26	15/57	Total mean	Tangency
29/42	8/53	77/89	53/89	213/61	Standard deviation	
0/22	0/35	-0/06	-0/04	0/07	Sharp ratio	

Table 7: Optimal Portfolios with Incremental recursive Estimation Period

Source: research findings

Table (7) reports the performance of exemplars of different strategies on average monthly returns, standard deviations and Sharp ratios of optimal portfolios with a recursive incremental estimation period. The highest average reported return is related to the tangent portfolio in a 24-month window. Most of the Sharp ratios relate to the mean variance strategy with the risk aversion coefficient of 5 and perform better than other strategies in all windows. The highest Sharp ratio was obtained in the 24-month window of the mean variance strategy (Risk Avoidance Coefficient 5). Also, the lowest standard deviation is related to the mean variance strategy with 5-risk aversion coefficient in the 24-month window.

Optimized portfolios with constant rolling estimation period

120 th window	90 th window	60 th window	48 th window	24 th window		Strategies
1/76	1/99	1/89	1/81	2	Total mean	Equal weight
4/74	4/50	4/26	4/22	4/27	Standard deviation	
0/37	0/44	0/44	0/43	0/47	Sharp ratio	
1/59	1/63	1/59	1/61	1/61	Total mean	Variance mean $\gamma = 3$
0/51	0/49	0/36	0/45	0/53	Standard deviation	
3/14	3/29	4/44	3/55	3/07	Sharp ratio	
1/56	1/56	1/56	1/57	1/57	Total mean	Variance mean $\gamma = 5$
0/36	0/33	0/24	0/27	0/32	Standard deviation	
4/38	4/08	6/53	5/7	4/92	Sharp ratio	
1/94	2/11	2/09	2/12	2/32	Total mean	Variance mean
5/30	4/62	4/55	4/57	4/26	Standard deviation	
0/37	0/46	0/46	0/46	0/54	Sharp ratio	
1/63	4/15	4/72	7/98	5/50	Total mean	Tangency
11/12	6/61	25/40	80/94	46/62	Standard deviation	
0/15	0/63	0/19	0/10	0/12	Sharp ratio	

Table 8. Optimized portfolios with constant rolling estimation period

Source: research finding

Table (8) reports the performance of a variety of strategies for monthly returns mean, standard deviations, and Sharp ratios of optimal portfolios with constant rolling estimation period. The highest reported average of return which is related to the tangent portfolio in a 48-month window. Most of the Sharp ratios were related to the mean variance strategy with the risk aversion coefficient of 5 and performed better than other strategies in all windows. The highest Sharp ratio was obtained in the 48- and 60-month windows. Also, the least standard deviation is related to the mean variance strategy with 5-risk aversion coefficient in the 60-month window.

Sharp Ratio Performance, Optimized Portfolios with Incremental Recursive Estimation Period

120 th window	90 th window	60 th window	48 th window	24 th window	strategies
0/37	0/44	0/44	0/43	0/47	$(\frac{1}{N})$
0/22 (0/14)	0/35 (0/26)	-00/06 (00/0)*	-0/04 (00/0)*	0/07 (00/0)*	Variance mean (Tangency)
2/73 (00/0)*	2/23 (00/0)*	2/21 (00/0)*	2/30 (00/0)*	3/66 (00/0)*	Variance mean $\gamma = 3$
3/97 (00/0)*	3/24 (00/0)*	3/24 (00/0)*	3/68 (00/0)*	5/94 (00/0)*	Variance mean $\gamma = 5$
0/37 0/4	0/43 0/38	0/43 0/38	0/42 0/4	0/47 (00/0)*	Minimum variance

Table 9- Sharp Ratio Performance of Different Strategies and Comparison with Strategy Performance $(\frac{1}{N})$

Soure: research findings

Table (9) contains the monthly Sharp Ratio of Strategy $(\frac{1}{N})$, the Extra Sharp Ratio of the mean variance strategy ($\gamma = 3, \gamma = 5$ and Tangency) and the Sharp Ratio of the minimum variance strategy. In parentheses, the P-value represents the difference between the Sharp Ratio of each strategy and the strategy $(\frac{1}{N})$ as the basis. The asterisk indicates the significant difference between the Sharp ratios of the two samples. Most of the Sharp ratios are related to the mean variance strategy with 5 risk aversion coefficients in all windows. The null hypothesis of the Sharp test is the absence of a significant difference between the Sharp ratio of each strategy and the strategy $(\frac{1}{N})$. According to the table, the mean variance strategy with risk aversion coefficients of 3 and 5 in all windows is able to defeat the strategy $(\frac{1}{N})$ in terms of Sharp ratios. Minimal variance strategy fails strategy $(\frac{1}{N})$ only in window 24. Minimal variance strategy fails strategy $(\frac{1}{N})$ only in window 24, in tangible portfolios, it is only able to defeat strategy $(\frac{1}{N})$ in the 24-month window and in other windows it cannot defeat strategy $(\frac{1}{N})$. In the 48- and 60-month window, the ratio of Sharp to Tangent Portfolios is negative, because in implementing this strategy when we normalize weights, the numerator in the denominator is a coincidence which is sometimes a small number (because of a very low risk averse number of people who invest in this type of portfolio), Therefore, in calculating weights, very large numbers are obtained, which results in very high or low returns, In other words, there is a sharp fluctuation in returns that ultimately results in a very low or negative Sharp ratio. It can be said that in this case it is not reasonable to invest in this type of portfolio and it is better to invest in a portfolio with a risk aversion factor. In general, the best performance of the mean variance strategy with both risk aversion coefficients of 3 and 5 is reported.

Sharp ratio performance, optimal portfolios with constant rolling estimation period

120 th window	90 th window	60 th window	48 th window	24 th window	strategies
0/37	0/44	0/44	0/43	0/47	$(\frac{1}{N})$
0/15 0/01	0/63 0/04	0/19 0/02	0/10 (00/0)*	0/12 (00/0)*	Variance mean (Tangency)
3/14 (00/0)*	3/29 (00/0)*	4/44 (00/0)*	3/55 (00/0)*	3/07 (00/0)*	Variance mean $\gamma = 3$
4/38 (00/0)*	4/80 (00/0)*	6/53 (00/0)*	5/70 (00/0)*	4/92 (00/0)*	Variance mean $5\gamma =$
0/37 (0/38)	0/46 (00/0)*	0/46 (0/35)	0/46 (0/19)	0/54 (0/15)	Minimum variance

Table 10- Sharp Ratio Performance of Different Strategies and Comparison with Strategy Performance $(\frac{1}{N})$

Source: research finding

Table (10) contains the monthly Sharp Ratio $(\frac{1}{N})$ ratio, the Extra Sharp Ratio of the mean variance strategy ($\gamma = 3, \gamma = 5$ and Tangency), and the Sharp Ratio of the minimum variance strategy in the rolling portfolios. In parentheses,

the P-Value represents the difference between the Sharp Ratio of each strategy and the strategy ($\frac{1}{N}$) as the basis. The asterisk indicates the significant difference between the Sharp ratios of the two samples. The highest Sharp ratios were obtained from the mean variance strategy with the risk aversion coefficient of 5 in all windows. The null hypothesis of the Sharp test is the absence of a significant difference between the Sharp ratio of each strategy and the strategy ($\frac{1}{N}$). According to the table, the mean variance strategy with risk aversion coefficients of 3 and 5 in all windows is able to defeat the strategy ($\frac{1}{N}$) in terms of Sharp ratios. Also in the 90 month window of minimal variance strategy is able to break strategy ($\frac{1}{N}$).

Conclusion

According to the results of the mean test, there was no significant difference between the mean of the strategies implemented and the strategy ($\frac{1}{N}$) in both return and rolling portfolios in all windows. In fact, none of the strategies can beat the average strategy ($\frac{1}{N}$). Also, according to the results of the variance comparison test, in both recursive and rolling portfolios in all windows, the mean variance strategy with risk aversion coefficients of 3 and 5 were able to defeat the strategy ($\frac{1}{N}$).

Sharp Ratio Test in Recurring Portfolio in Averaged Avoidance Strategy with Risk Avoidance (3,5) in All Windows and Minimum Variance Strategy Only in Window 24 Able to Defeat Strategy ($\frac{1}{N}$) based on sharp ratio. Also, tangent portfolios in 24-month windows are also able to break strategy ($\frac{1}{N}$). In rolling portfolios, the mean variance strategy with risk aversion coefficient (3,5) is able to defeat the strategy ($\frac{1}{N}$) in terms of Sharp ratio. But in the tangent portfolio only in the 24 and 48 window and in the minimum variance strategy in the 90 month strategy window ($\frac{1}{N}$) fail. According to the results, it can be concluded that the performance of rolling portfolios and return strategy with mean variance strategy with specified risk-aversion coefficient in all windows is able to defeat the strategy ($\frac{1}{N}$). But the tangible portfolio and the portfolio based on the Sharp Ratio strategy performed poorly in most windows. The reason for the better performance of the mean variance strategy is that the underlying decision making is the predictability of asset returns, and the weighting of each asset is based on the predicted maximum return per month.

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